Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced

Tuesday 16 June 2015 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. (*a*) Find the binomial expansion of

$$(4+5x)^{\frac{1}{2}}, \quad |x| < \frac{4}{5},$$

in ascending powers of x, up to and including the term in x^2.
Give each coefficient in its simplest form.

(5)

(*b*) Find the exact value of $(4+5x)^{\frac{1}{2}}$ when $x = \frac{1}{10}$.

Give your answer in the form $k\sqrt{2}$, where k is a constant to be determined.

(1)

(*c*) Substitute $x = \frac{1}{10}$ into your binomial expansion from part (*a*) and hence find an approximate value for $\sqrt{2}$.

Give your answer in the form $\frac{p}{q}$, where p and q are integers.

(2)

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2. The curve C has equation

$$x^2 - 3xy - 4y^2 + 64 = 0.$$

(*a*) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y.

(5)

(*b*) Find the coordinates of the points on C where $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

3.

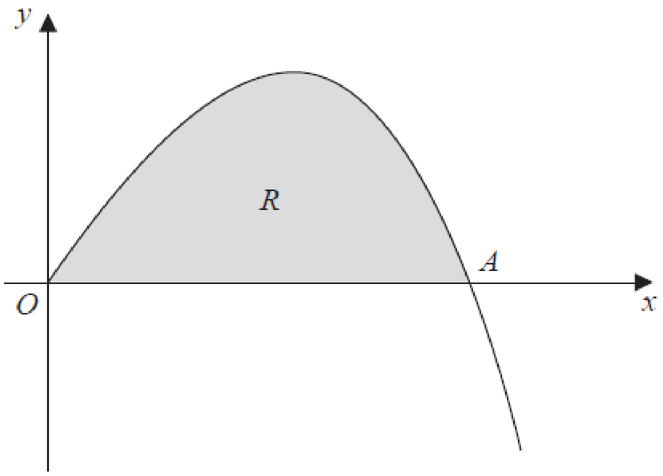

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = 4x - xe^{\frac{1}{2}x}$, $x \geq 0$.

The curve meets the x-axis at the origin O and cuts the x-axis at the point A.

(*a*) Find, in terms of ln 2, the x coordinate of the point A.

(2)

(*b*) Find $\displaystyle\int xe^{\frac{1}{2}x}\,dx$.

(3)

The finite region R, shown shaded in Figure 1, is bounded by the x-axis and the curve with equation $y = 4x - xe^{\frac{1}{2}x}$, $x \geq 0$.

(*c*) Find, by integration, the exact value for the area of R.
Give your answer in terms of ln 2.

(3)

4. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1\text{: } \mathbf{r} = \begin{pmatrix} 5 \\ -3 \\ p \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}, \qquad l_2\text{: } \mathbf{r} = \begin{pmatrix} 8 \\ 5 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix},$$

where λ and μ are scalar parameters and p is a constant.

The lines l_1 and l_2 intersect at the point A.

(*a*) Find the coordinates of A.

(2)

(*b*) Find the value of the constant p.

(3)

(*c*) Find the acute angle between l_1 and l_2, giving your answer in degrees to 2 decimal places.

(3)

The point B lies on l_2 where $\mu = 1$.

(*d*) Find the shortest distance from the point B to the line l_1, giving your answer to 3 significant figures.

(3)

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5. A curve C has parametric equations

$$x = 4t + 3, \qquad y = 4t + 8 + \frac{5}{2t}, \qquad t \neq 0.$$

(*a*) Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point on C where $t = 2$, giving your answer as a fraction in its simplest form.

(3)

(*b*) Show that the cartesian equation of the curve C can be written in the form

$$y = \frac{x^2 + ax + b}{x - 3}, \qquad x \neq 3,$$

where a and b are integers to be determined.

(3)

6.

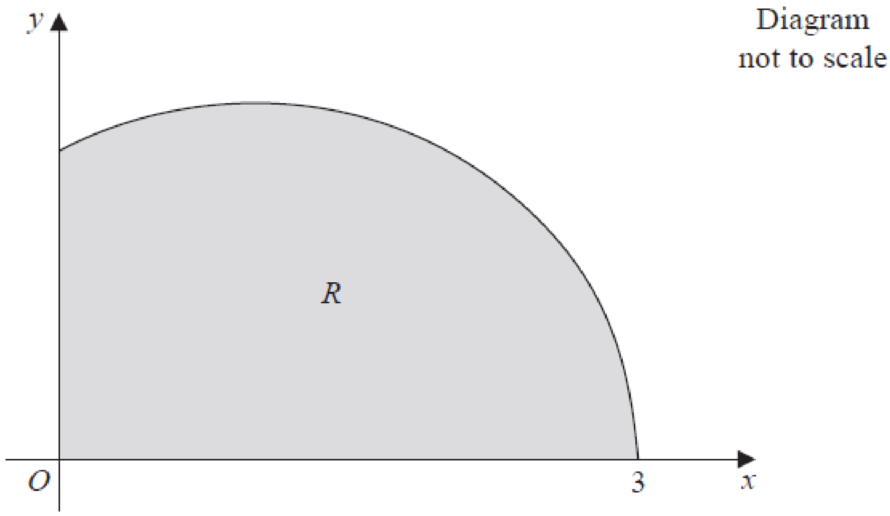

Figure 2

Figure 2 shows a sketch of the curve with equation $y = \sqrt{(3-x)(x+1)}$, $0 \leq x \leq 3$.

The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis, and the y-axis.

(a) Use the substitution $x = 1 + 2\sin\theta$ to show that

$$\int_0^3 \sqrt{(3-x)(x+1)}\ \mathrm{d}x = k\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta\ \mathrm{d}\theta,$$

where k is a constant to be determined.

(5)

(b) Hence find, by integration, the exact area of R.

(3)

7. (*a*) Express $\dfrac{2}{P(P-2)}$ in partial fractions.

(3)

A team of biologists is studying a population of a particular species of animal.

The population is modelled by the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{1}{2}P(P-2)\cos 2t, \qquad t \geq 0,$$

where P is the population in thousands, and t is the time measured in years since the start of the study.

Given that $P = 3$ when $t = 0$,

(*b*) solve this differential equation to show that

$$P = \frac{6}{3 - e^{\frac{1}{2}\sin 2t}}$$

(7)

(*c*) find the time taken for the population to reach 4000 for the first time.
Give your answer in years to 3 significant figures.

(3)

8.

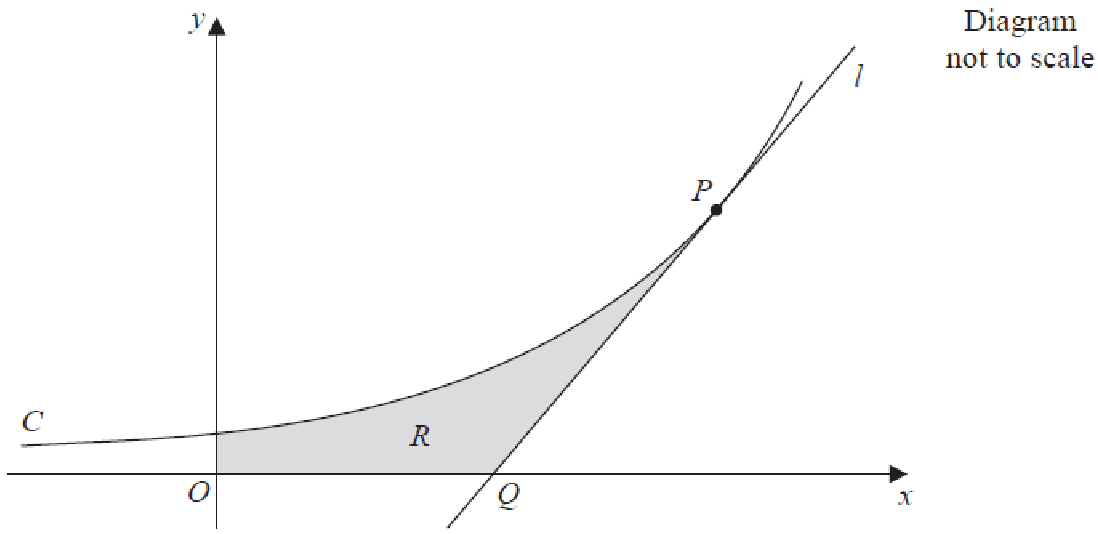

Figure 3

Figure 3 shows a sketch of part of the curve C with equation $y = 3^x$.

The point P lies on C and has coordinates (2, 9).

The line l is a tangent to C at P. The line l cuts the x-axis at the point Q.

(a) Find the exact value of the x coordinate of Q.

(4)

The finite region R, shown shaded in Figure 3, is bounded by the curve C, the x-axis, the y-axis and the line l. This region R is rotated through 360° about the x-axis.

(b) Use integration to find the exact value of the volume of the solid generated.

Give your answer in the form $\frac{p}{q}$, where p and q are exact constants.

[*You may assume the formula* $V = \frac{1}{3}\pi r^2 h$ *for the volume of a cone.*]

(6)

TOTAL FOR PAPER: 75 MARKS

END

June 2015
6666/01 Core Mathematics 4
Mark Scheme

Question Number	Scheme	Marks
1. (a)	$(4+5x)^{\frac{1}{2}} = \underline{(4)^{\frac{1}{2}}}\left(1+\frac{5x}{4}\right)^{\frac{1}{2}} = \underline{2}\left(1+\frac{5x}{4}\right)^{\frac{1}{2}}$ $\underline{(4)^{\frac{1}{2}}}$ or $\underline{2}$	B1
	$=\{2\}\left[1+\left(\frac{1}{2}\right)(kx)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2+\dots\right]$ see notes	M1 A1ft
	$=\{2\}\left[1+\left(\frac{1}{2}\right)\left(\frac{5x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(\frac{5x}{4}\right)^2+\dots\right]$ $=2\left[1+\frac{5}{8}x-\frac{25}{128}x^2+\dots\right]$ See notes below!	
	$=2+\frac{5}{4}x;-\frac{25}{64}x^2+\dots$ isw	A1; A1
		[5]
(b)	$\left\{x=\frac{1}{10}\Rightarrow(4+5(0.1))^{\frac{1}{2}}=\sqrt{4.5}=\sqrt{\frac{9}{2}}=\underline{\underline{\frac{3}{\sqrt{2}}}}=\frac{3}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}}\right\}$	
	$=\frac{3}{2}\sqrt{2}$ $\frac{3}{2}\sqrt{2}$ or $k=\frac{3}{2}$ or 1.5 o.e.	B1
		[1]
(c)	$\frac{3}{2}\sqrt{2}$ or $1.5\sqrt{2}$ or $\underline{\underline{\frac{3}{\sqrt{2}}}}=2+\frac{5}{4}\left(\frac{1}{10}\right)-\frac{25}{64}\left(\frac{1}{10}\right)^2+\dots\ \{=2.121\dots\}$ See notes	M1
	So, $\frac{3}{2}\sqrt{2}=\frac{543}{256}$ or $\underline{\underline{\frac{3}{\sqrt{2}}}}=\frac{543}{256}$	
	yields, $\sqrt{2}=\frac{181}{128}$ or $\sqrt{2}=\frac{256}{181}$ $\frac{181}{128}$ or $\frac{362}{256}$ or $\frac{543}{384}$ **or** $\frac{256}{181}$ etc.	A1 oe
		[2] **8**

	Question 1 Notes	
1. (a)	**B1**	$\underline{(4)^{\frac{1}{2}}}$ or $\underline{2}$ outside brackets or $\underline{2}$ as candidate's constant term in their binomial expansion.
	M1	Expands $(\dots+kx)^{\frac{1}{2}}$ to give any 2 terms out of 3 terms simplified or un-simplified, Eg: $1+\left(\frac{1}{2}\right)(kx)$ or $\left(\frac{1}{2}\right)(kx)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2$ or $1+\dots+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2$ where k is a numerical value and **where** $k\neq 1$.
	A1	A correct simplified or un-simplified $1+\left(\frac{1}{2}\right)(kx)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2$ expansion with **consistent** (kx).
	Note	(kx), $k\neq 1$, must be consistent (on the RHS, not necessarily on the LHS) in a candidate's expansion.

1. (a) ctd.	**Note**	Award B1M1A0 for $2\left[1+\left(\frac{1}{2}\right)(5x)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(\frac{5x}{4}\right)^2+...\right]$ because (kx) is not consistent.
	Note	**<u>Incorrect bracketing:</u>** $2\left[1+\left(\frac{1}{2}\right)\left(\frac{5x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(\frac{5x^2}{4}\right)+...\right]$ is B1M1A0 unless recovered.
	A1	$2+\frac{5}{4}x$ **(simplified fractions)** or allow $2+1.25x$ or $2+1\frac{1}{4}x$
	A1	Accept only $-\frac{25}{64}x^2$ or $-0.390625x^2$
	SC	If a candidate ***would otherwise score*** 2nd A0, 3rd A0 then **allow Special Case 2nd A1 for either** **SC:** $2\left[1+\frac{5}{8}x; ...\right]$ or **SC:** $2\left[1+...-\frac{25}{128}x^2+...\right]$ or **SC**: $\lambda\left[1+\frac{5}{8}x-\frac{25}{128}x^2+...\right]$ or **SC**: $\left[\lambda+\frac{5\lambda}{8}x-\frac{25\lambda}{128}x^2+...\right]$ (where λ can be 1 or omitted), where each term in the $[.....]$ is a simplified fraction or a decimal, **OR SC:** for $2+\frac{10}{8}x-\frac{50}{128}x^2+...$ (i.e. for not simplifying their correct coefficients.)
	Note	Candidates who write $2\left[1+\left(\frac{1}{2}\right)\left(-\frac{5x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(-\frac{5x}{4}\right)^2+...\right]$, where $k=-\frac{5}{4}$ and not $\frac{5}{4}$ and achieve $2-\frac{5}{4}x-\frac{25}{64}x^2+...$ will get B1M1A1A0A1
	Note	Ignore extra terms beyond the term in x^2.
	Note	You can ignore subsequent working following a correct answer.
(b)	**B1**	$\frac{3}{2}\sqrt{2}$ or $1.5\sqrt{2}$ or $k=\frac{3}{2}$ or 1.5 o.e. (Ignore how $k=\frac{3}{2}$ is found.)
(c)	**M1**	Substitutes $x=\frac{1}{10}$ or 0.1 into their binomial expansion found in part (a) which must contain both an x term and an x^2 term (or even an x^3 term) **and** equates this to either $\frac{3}{\sqrt{2}}$ or their $k\sqrt{2}$ from (b), where k is a numerical value.
	Note	M1 can be implied by $\frac{3}{2}\sqrt{2}$ or $1.5\sqrt{2}$ or $\frac{3}{\underline{\underline{\sqrt{2}}}}$ = awrt 2.121
	Note	M1 ***can be implied*** by $\frac{1}{k}\left(\text{their } \frac{543}{256}\right)$, with their k found in part (b).
	Note	M1 ***cannot be implied*** by $(k)\left(\text{their } \frac{543}{256}\right)$, with their k found in part (b).
	A1	$\frac{181}{128}$ **or any equivalent fraction**, eg: $\frac{362}{256}$ or $\frac{543}{384}$. Also allow $\frac{256}{181}$ **or any equivalent fraction.**
	Note	Also allow A1 for $p=181, q=128$ or $p=181\lambda, q=128\lambda$ or $p=256, q=181$ or $p=256\lambda, q=181\lambda$, where $\lambda\in\mathbb{Z}^+$
	Note	You can recover work for part (c) in part (b). You cannot recover part (b) work in part (c).
	Note	Candidates are allowed to restart and gain all 2 marks in part (c) from an incorrect part (b).
	Note	Award M1 A1 for the correct answer from no working.

1. (a)	**Alternative methods for part (a)**	
	Alternative method 1: Candidates can apply an alternative form of the binomial expansion.	
	$\left\{(4+5x)^{\frac{1}{2}}\right\} = (4)^{\frac{1}{2}} + \left(\tfrac{1}{2}\right)(4)^{-\frac{1}{2}}(5x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(4)^{-\frac{3}{2}}(5x)^2$	
	B1	$(4)^{\frac{1}{2}}$ or 2
	M1	Any two of three (un-simplified) terms correct.
	A1	All three (un-simplified) terms correct.
	A1	$2+\frac{5}{4}x$ **(simplified fractions)** or allow $2+1.25x$ or $2+1\frac{1}{4}x$
	A1	Accept only $-\frac{25}{64}x^2$ or $-0.390625x^2$
	Note	The terms in C need to be evaluated. So ${}^{\frac{1}{2}}C_0(4)^{\frac{1}{2}} + {}^{\frac{1}{2}}C_1(4)^{-\frac{1}{2}}(5x); + {}^{\frac{1}{2}}C_2(4)^{-\frac{3}{2}}(5x)^2$ without further working is B0M0A0.

Alternative Method 2: Maclaurin Expansion $f(x) = (4+5x)^{\frac{1}{2}}$

$f''(x) = -\frac{25}{4}(4+5x)^{-\frac{3}{2}}$	Correct $f''(x)$	B1
$f'(x) = \frac{1}{2}(4+5x)^{-\frac{1}{2}}(5)$	$\pm a(4+5x)^{-\frac{1}{2}}$; $a \neq \pm 1$	M1
	$\frac{1}{2}(4+5x)^{-\frac{1}{2}}(5)$	A1 oe
$\left\{\therefore f(0)=2,\ f'(0)=\frac{5}{4} \text{ and } f''(0)=-\frac{25}{32}\right\}$		
So, $f(x) = 2+\frac{5}{4}x; -\frac{25}{64}x^2+\ldots$		A1; A1

Question Number	Scheme	Marks
2.	$x^2 - 3xy - 4y^2 + 64 = 0$	
(a)	$\left\{\frac{\cancel{dy}}{\cancel{dx}} \cancel{=}\right\}\quad \underline{2x} - \left(\underline{\underline{3y + 3x\frac{dy}{dx}}}\right) \underline{- 8y\frac{dy}{dx}} = \underline{0}$	M1$\underline{\text{A1}}$ $\underline{\underline{\text{M1}}}$
	$2x - 3y + (-3x - 8y)\frac{dy}{dx} = 0$	dM1
	$\frac{dy}{dx} = \frac{2x-3y}{3x+8y}$ or $\frac{3y-2x}{-3x-8y}$ **o.e.**	A1 **cso**
		[5]
(b)	$\left\{\frac{dy}{dx} = 0 \Rightarrow\right\}\ 2x - 3y = 0$	M1
	$y = \frac{2}{3}x$ | $x = \frac{3}{2}y$	A1ft
	$x^2 - 3x\left(\frac{2}{3}x\right) - 4\left(\frac{2}{3}x\right)^2 + 64 = 0$ | $\left(\frac{3}{2}y\right)^2 - 3\left(\frac{3}{2}y\right)y - 4y^2 + 64 = 0$	dM1
	$x^2 - 2x^2 - \frac{16}{9}x^2 + 64 = 0 \Rightarrow -\frac{25}{9}x^2 + 64 = 0$ | $\frac{9}{4}y^2 - \frac{9}{2}y^2 - 4y^2 + 64 = 0 \Rightarrow -\frac{25}{4}y^2 + 64 = 0$	
	$\left\{\Rightarrow x^2 = \frac{576}{25} \Rightarrow\right\}\ x = \frac{24}{5}$ or $-\frac{24}{5}$ | $\left\{\Rightarrow y^2 = \frac{256}{25} \Rightarrow\right\}\ y = \frac{16}{5}$ or $-\frac{16}{5}$	A1 **cso**
	When $x = \pm\frac{24}{5}$, $y = \frac{2}{3}\left(\frac{24}{5}\right)$ and $-\frac{2}{3}\left(\frac{24}{5}\right)$ | When $y = \pm\frac{16}{5}$, $x = \frac{3}{2}\left(\frac{16}{5}\right)$ and $-\frac{3}{2}\left(\frac{16}{5}\right)$	
	$\left(\frac{24}{5}, \frac{16}{5}\right)$ **and** $\left(-\frac{24}{5}, -\frac{16}{5}\right)$ **or** $x = \frac{24}{5}$, $y = \frac{16}{5}$ **and** $x = -\frac{24}{5}$, $y = -\frac{16}{5}$ **cso**	ddM1 A1
		[6] **11**
	Alternative method for part (a)	
(a)	$\left\{\frac{\cancel{dx}}{\cancel{dy}} \cancel{=}\right\}\quad \underline{2x\frac{dx}{dy}} - \left(\underline{\underline{3y\frac{dx}{dy} + 3x}}\right) \underline{- 8y} = \underline{0}$	M1$\underline{\text{A1}}$ $\underline{\underline{\text{M1}}}$
	$(2x - 3y)\frac{dx}{dy} - 3x - 8y = 0$	dM1
	$\frac{dy}{dx} = \frac{2x-3y}{3x+8y}$ or $\frac{3y-2x}{-3x-8y}$ **o.e.**	A1 **cso**
		[5]

	Question 2 Notes	
2. (a) **General**	**Note**	Writing down $\frac{dy}{dx} = \frac{2x-3y}{3x+8y}$ or $\frac{3y-2x}{-3x-8y}$ from **no working** is full marks
	Note	Writing down $\frac{dy}{dx} = \frac{2x-3y}{-3x-8y}$ or $\frac{3y-2x}{3x+8y}$ from **no working** is M1A0B1M1A0
	Note	Few candidates will write $2x\,dx - 3y\,dx - 3x\,dy - 8y\,dy = 0$ leading to $\frac{dy}{dx} = \frac{2x-3y}{3x+8y}$, o.e. This should get full marks.

2. (a)	**M1**	Differentiates implicitly to include either $\pm 3x\frac{\mathrm{d}y}{\mathrm{d}x}$ or $-4y^2 \to \pm ky\frac{\mathrm{d}y}{\mathrm{d}x}$. (Ignore $\left(\frac{\mathrm{d}y}{\mathrm{d}x} = \right)$).
	A1	**Both** $x^2 \to \underline{2x}$ **and** $\ldots - 4y^2 + 64 = 0 \to \underline{-8y\frac{\mathrm{d}y}{\mathrm{d}x} = 0}$
	Note	If an extra term appears then award A0.
	M1	$-3xy \to -3x\frac{\mathrm{d}y}{\mathrm{d}x} - 3y$ or $-3x\frac{\mathrm{d}y}{\mathrm{d}x} + 3y$ or $3x\frac{\mathrm{d}y}{\mathrm{d}x} - 3y$ or $3x\frac{\mathrm{d}y}{\mathrm{d}x} + 3y$
	Note	$2x - 3y - 3x\frac{\mathrm{d}y}{\mathrm{d}x} - 8y\frac{\mathrm{d}y}{\mathrm{d}x} \to 2x - 3y = 3x\frac{\mathrm{d}y}{\mathrm{d}x} + 8y\frac{\mathrm{d}y}{\mathrm{d}x}$ will get 1st A1 (implied) as the "$= 0$" can be implied by the rearrangement of their equation.
	dM1	**dependent on the FIRST method mark being awarded.** An attempt to factorise out **all the terms in** $\frac{\mathrm{d}y}{\mathrm{d}x}$ as long as there are ***at least two terms*** in $\frac{\mathrm{d}y}{\mathrm{d}x}$. i.e. $\ldots + (-3x - 8y)\frac{\mathrm{d}y}{\mathrm{d}x} = \ldots$ or $\ldots = (3x + 8y)\frac{\mathrm{d}y}{\mathrm{d}x}$. (Allow combining in 1 variable).
	A1	$\frac{2x - 3y}{3x + 8y}$ or $\frac{3y - 2x}{-3x - 8y}$ or equivalent.
	Note	**cso** If the candidate's solution is not completely correct, then do not give this mark.
	Note	You cannot recover work for part (a) in part (b).
2. (b)	**M1**	Sets their numerator of their $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to zero (or the denominator of their $\frac{\mathrm{d}x}{\mathrm{d}y}$ equal to zero) o.e.
	Note	1st M1 can also be gained by setting $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to zero in their "$2x - 3y - 3x\frac{\mathrm{d}y}{\mathrm{d}x} - 8y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$"
	Note	**If their numerator involves one variable only then only the 1st M1 mark is possible in part (b).**
	Note	**If their numerator is a constant then no marks are available in part (b)**
	Note	**If their numerator is in the form** $\pm ax^2 \pm by = 0$ or $\pm ax \pm by^2 = 0$ **then the first 3 marks are possible in part (b).**
	Note	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x - 3y}{3x + 8y} = 0$ is not sufficient for M1.
	A1ft	Either • Sets $2x - 3y$ to zero and obtains either $y = \frac{2}{3}x$ or $x = \frac{3}{2}y$ • the follow through result of making either *y* or *x* the subject from setting their numerator of their $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to zero
	dM1	**dependent on the first method mark being awarded.** Substitutes ***either*** their $y = \frac{2}{3}x$ ***or*** their $x = \frac{3}{2}y$ into the original equation to give an equation in one variable only.
	A1	Obtains either $x = \frac{24}{5}$ or $-\frac{24}{5}$ or $y = \frac{16}{5}$ or $-\frac{16}{5}$, (or equivalent) ***by correct solution only***. i.e. You can allow for example $x = \frac{48}{10}$ or 4.8, etc.
	Note	$x = \sqrt{\frac{576}{25}}$ (not simplified) or $y = \sqrt{\frac{256}{25}}$ (not simplified) is not sufficient for A1.

2. (b) ctd	**ddM1**	**dependent on both previous method marks being awarded in this part.** **<u>Method 1</u>** **Either:** • substitutes their x into their $y = \frac{2}{3}x$ **or** substitutes their y into their $x = \frac{3}{2}y$, **or** • substitutes ***the other of*** their $y = \frac{2}{3}x$ ***or*** their $x = \frac{3}{2}y$ into the original equation, **and achieves either:** • exactly two sets of two coordinates **or** • exactly two distinct values for x and exactly two distinct values for y. **<u>Method 2</u>** **Either:** • substitutes their first x-value, x_1 into $x^2 - 3xy - 4y^2 + 64 = 0$ to obtain one y-value, y_1 and substitutes their second x-value, x_2 into $x^2 - 3xy - 4y^2 + 64 = 0$ to obtain 1 y-value y_2 **or** • substitutes their first y-value, y_1 into $x^2 - 3xy - 4y^2 + 64 = 0$ to obtain one x-value x_1 and substitutes their second y-value, y_2 into $x^2 - 3xy - 4y^2 + 64 = 0$ to obtain one x-value x_2.
	Note	Three or more sets of coordinates given (without identification of two sets of coordinates) is ddM0.
	A1	Both $\left(\frac{24}{5}, \frac{16}{5}\right)$ **and** $\left(-\frac{24}{5}, -\frac{16}{5}\right)$, only **by cso.** Note that decimal equivalents are fine.
	Note	Also allow $x = \frac{24}{5}$, $y = \frac{16}{5}$ **and** $x = -\frac{24}{5}$, $y = -\frac{16}{5}$ all seen in their working to part (b).
	Note	Allow $x = \pm\frac{24}{5}$, $y = \pm\frac{16}{5}$ for 3rd A1.
	Note	$x = \pm\frac{24}{5}$, $y = \pm\frac{16}{5}$ followed by eg. $\left(\frac{16}{5}, \frac{24}{5}\right)$ **and** $\left(-\frac{16}{5}, -\frac{24}{5}\right)$ (eg. coordinates stated the wrong way round) is 3rd A0.
	Note	It is possible for a candidate who does not achieve full marks in part (a), (but has a correct numerator for $\frac{dy}{dx}$) to gain all 6 marks in part (b).
	Note	Decimal equivalents to fractions are fine in part (b). i.e. $(4.8, 3.2)$ **and** $(-4.8, -3.2)$.
	Note	$\left(\frac{24}{5}, \frac{16}{5}\right)$ **and** $\left(-\frac{24}{5}, -\frac{16}{5}\right)$ from no working is M0A0M0A0M0A0.
	Note	Candidates could potentially lose the final 2 marks for setting both their numerator and denominator to zero.
	Note	No credit in this part can be gained by only setting the denominator to zero.

Question Number	Scheme		Marks
3.	$y = 4x - x\mathrm{e}^{\frac{1}{2}x},\ x \geqslant 0$		
(a)	$\left\{y = 0 \Rightarrow 4x - x\mathrm{e}^{\frac{1}{2}x} = 0 \Rightarrow x(4 - \mathrm{e}^{\frac{1}{2}x}) = 0 \Rightarrow\right\}$		
	$\mathrm{e}^{\frac{1}{2}x} = 4 \Rightarrow x_A = 4\ln 2$	Attempts to solve $\mathrm{e}^{\frac{1}{2}x} = 4$ giving $x = ...$ in terms of $\pm\lambda\ln\mu$ where $\mu > 0$	M1
		$4\ln 2$ **cao** (Ignore $x = 0$)	A1
			[2]
(b)	$\left\{\int x\mathrm{e}^{\frac{1}{2}x}\,\mathrm{d}x\right\} = 2x\mathrm{e}^{\frac{1}{2}x} - \int 2\mathrm{e}^{\frac{1}{2}x}\{\mathrm{d}x\}$	$\alpha x\mathrm{e}^{\frac{1}{2}x} - \beta\int \mathrm{e}^{\frac{1}{2}x}\{\mathrm{d}x\}$, $\alpha > 0, \beta > 0$	M1
		$2x\mathrm{e}^{\frac{1}{2}x} - \int 2\mathrm{e}^{\frac{1}{2}x}\{\mathrm{d}x\}$, with or without $\mathrm{d}x$	A1 **(M1 on ePEN)**
	$= 2x\mathrm{e}^{\frac{1}{2}x} - 4\mathrm{e}^{\frac{1}{2}x}\ \{+c\}$	$2x\mathrm{e}^{\frac{1}{2}x} - 4\mathrm{e}^{\frac{1}{2}x}$ o.e. with or without $+c$	A1
			[3]
(c)	$\left\{\int 4x\,\mathrm{d}x\right\} = 2x^2$	$4x \to 2x^2$ or $\dfrac{4x^2}{2}$ o.e.	B1
	$\left\{\int_0^{4\ln 2}(4x - x\mathrm{e}^{\frac{1}{2}x})\,\mathrm{d}x\right\} = \left[2x^2 - \left(2x\mathrm{e}^{\frac{1}{2}x} - 4\mathrm{e}^{\frac{1}{2}x}\right)\right]_0^{4\ln 2 \text{ or } \ln 16 \text{ or their limits}}$		
	$= \left(2(4\ln 2)^2 - 2(4\ln 2)\mathrm{e}^{\frac{1}{2}(4\ln 2)} + 4\mathrm{e}^{\frac{1}{2}(4\ln 2)}\right) - \left(2(0)^2 - 2(0)\mathrm{e}^{\frac{1}{2}(0)} + 4\mathrm{e}^{\frac{1}{2}(0)}\right)$	**See notes**	M1
	$= \left(32(\ln 2)^2 - 32(\ln 2) + 16\right) - (4)$ $= 32(\ln 2)^2 - 32(\ln 2) + 12$	$32(\ln 2)^2 - 32(\ln 2) + 12$, **see notes**	A1
			[3] **8**

		Question 3 Notes
3. (a)	**M1**	Attempts to solve $\mathrm{e}^{\frac{1}{2}x} = 4$ giving $x = ...$ in terms of $\pm\lambda\ln\mu$ where $\mu > 0$
	A1	$4\ln 2$ **cao** stated in part (a) only (Ignore $x = 0$)
(b)	**NOTE**	**Part (b) appears as M1M1A1 on ePEN, but is now marked as M1A1A1.**
	M1	Integration by parts is applied in the form $\alpha x\mathrm{e}^{\frac{1}{2}x} - \beta\int \mathrm{e}^{\frac{1}{2}x}\{\mathrm{d}x\}$, where $\alpha > 0, \beta > 0$. (must be in this form) with or without $\mathrm{d}x$
	A1	$2x\mathrm{e}^{\frac{1}{2}x} - \int 2\mathrm{e}^{\frac{1}{2}x}\{\mathrm{d}x\}$ or equivalent, with or without $\mathrm{d}x$. **Can be un-simplified.**
	A1	$2x\mathrm{e}^{\frac{1}{2}x} - 4\mathrm{e}^{\frac{1}{2}x}$ or equivalent with or without + c. **Can be un-simplified.**
	Note	You can also allow $2\mathrm{e}^{\frac{1}{2}x}(x - 2)$ or $\mathrm{e}^{\frac{1}{2}x}(2x - 4)$ for the final A1.
	isw	You can ignore subsequent working following on from a correct solution.
	SC	**<u>SPECIAL CASE:</u>** A candidate who uses $u = x$, $\dfrac{\mathrm{d}v}{\mathrm{d}x} = \mathrm{e}^{\frac{1}{2}x}$, writes down the correct "by parts" formula, but makes only one error when applying it can be awarded Special Case M1. (Applying their v counts for one consistent error.)

3. (c)	**B1**	$4x \to 2x^2$ or $\frac{4x^2}{2}$ oe
	M1	**Complete** method of applying limits of their x_A and 0 to all terms of an expression of the form $\pm Ax^2 \pm Bx\mathrm{e}^{\frac{1}{2}x} \pm C\mathrm{e}^{\frac{1}{2}x}$ (where $A \neq 0$, $B \neq 0$ and $C \neq 0$) and subtracting the correct way round.
	Note	Evidence of a proper consideration of the limit of 0 is needed for M1. So subtracting 0 is M0.
	Note	$\ln 16$ or $2\ln 4$ or equivalent is fine as an upper limit.
	A1	A correct three term exact quadratic expression in $\ln 2$. For example allow for A1 • $32(\ln 2)^2 - 32(\ln 2) + 12$ • $8(2\ln 2)^2 - 8(4\ln 2) + 12$ • $2(4\ln 2)^2 - 32(\ln 2) + 12$ • $2(4\ln 2)^2 - 2(4\ln 2)\mathrm{e}^{\frac{1}{2}(4\ln 2)} + 12$
	Note	Note that the constant term of 12 needs to be combined from $4\mathrm{e}^{\frac{1}{2}(4\ln 2)} - 4\mathrm{e}^{\frac{1}{2}(0)}$ o.e.
	Note	Also allow $32\ln 2(\ln 2 - 1) + 12$ or $32\ln 2\left(\ln 2 - 1 + \frac{12}{32\ln 2}\right)$ for A1.
	Note	***Do not apply "ignore subsequent working" for incorrect simplification.*** Eg: $32(\ln 2)^2 - 32(\ln 2) + 12 \to 64(\ln 2) - 32(\ln 2) + 12$ or $32(\ln 4) - 32(\ln 2) + 12$
	Note	**Bracketing error:** $32\ln 2^2 - 32(\ln 2) + 12$, unless recovered is final A0.
	Note	**Notation:** Allow $32(\ln^2 2) - 32(\ln 2) + 12$ for the final A1.
	Note	5.19378… without seeing $32(\ln 2)^2 - 32(\ln 2) + 12$ is A0.
	Note	5.19378… following from a correct $2x^2 - \left(2x\mathrm{e}^{\frac{1}{2}x} - 4\mathrm{e}^{\frac{1}{2}x}\right)$ is M1A0.
	Note	5.19378… from no working is M0A0.

Question Number	Scheme		Marks
4.	$l_1 : \mathbf{r} = \begin{pmatrix} 5 \\ -3 \\ p \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}$, $l_2 : \mathbf{r} = \begin{pmatrix} 8 \\ 5 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$. Let $\theta =$ acute angle between l_1 and l_2. **Note: You can mark parts (a) and (b) together.**		
(a)	$\{l_1 = l_2 \Rightarrow \mathbf{i}:\}\ 5 = 8 + 3\mu \Rightarrow \mu = -1$	Finds μ and substitutes their μ into l_2	M1
	So, $\{\overrightarrow{OA}\} = \begin{pmatrix} 8 \\ 5 \\ -2 \end{pmatrix} - 1 \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$	$5\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ or $\begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$ or $(5, 1, 3)$	A1
			[2]
(b)	$\{\mathbf{j}:\ -3 + \lambda = 5 + 4\mu \Rightarrow\}\ -3 + \lambda = 5 + 4(-1) \Rightarrow \lambda = 4$	Equates **j** components, substitutes their μ and solves to give $\lambda = ...$	M1
	$\mathbf{k}:\ p - 3\lambda = -2 - 5\mu \Rightarrow$ $p - 3(4) = -2 - 5(-1) \Rightarrow \underline{p = 15}$ or $\mathbf{k}:\ p - 3\lambda = 3 \Rightarrow$ $p - 3(4) = 3 \Rightarrow \underline{p = 15}$	Equates **k** components, substitutes their λ and their μ and solves to give $p = ...$ **or** equates **k** components to give their "$p - 3\lambda =$ the **k** value of A found in part (a)", substitutes their λ and solves to give $p = ...$	M1
		$p = 15$	A1
			[3]
(c)	$\mathbf{d_1} = \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}$, $\mathbf{d}_2 = \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$	Realisation that the dot product is required between $\pm A\mathbf{d_1}$ and $\pm B\mathbf{d}_2$.	M1
	$\cos\theta = \pm K\left(\dfrac{0(3) + (1)(4) + (-3)(-5)}{\sqrt{(0)^2 + (1)^2 + (-3)^2}\,.\sqrt{(3)^2 + (4)^2 + (-5)^2}}\right)$	An attempt to apply the dot product formula between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$.	dM1 **(A1 on ePEN)**
	$\cos\theta = \dfrac{19}{\sqrt{10}.\sqrt{50}} \Rightarrow \theta = 31.8203116... = 31.82$ (2 dp)	anything that rounds to 31.82	A1
			[3]
(d)	$\overrightarrow{OB} = \begin{pmatrix} 11 \\ 9 \\ -7 \end{pmatrix}$; $\overrightarrow{AB} = \begin{pmatrix} 11 \\ 9 \\ -7 \end{pmatrix} - \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \\ -10 \end{pmatrix}$ or $\overrightarrow{AB} = 2\begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \\ -10 \end{pmatrix}$ $\lvert\overrightarrow{AB}\rvert = \sqrt{6^2 + 8^2 + (-10)^2}\ \{= 10\sqrt{2}\}$	**See notes**	M1
	$\dfrac{d}{10\sqrt{2}} = \sin\theta$	Writes down a correct trigonometric equation involving the shortest distance, d. Eg: $\dfrac{d}{\text{their } AB} = \sin\theta$, oe.	dM1
	$\{d = 10\sqrt{2}\sin 31.82... \Rightarrow\}\ d = 7.456540753... = 7.46$ (3sf)	anything that rounds to 7.46	A1
			[3] **11**

4. (b)	**Alternative method for part (b)**		
	$\left\{\begin{array}{l} 3\times\mathbf{j}: \ -9+3\lambda = 15+12\mu \\ \mathbf{k}: \ \ p-3\lambda = -2+5\mu \end{array}\right\}$ $p-9=13+7\mu$	Eliminates λ to write down an equation in p and μ	M1
	$p-9=13+7(-1) \Rightarrow \underline{p=15}$	Substitutes their μ and solves to give $p = ...$	M1
		$p=15$	A1
4. (d)	**Alternative Methods for part (d)** Let X be the foot of the perpendicular from B onto l_1		
	$\mathbf{d}_1 = \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}$, $\overrightarrow{OX} = \begin{pmatrix} 5 \\ -3 \\ 15 \end{pmatrix} + \lambda\begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 5 \\ -3+\lambda \\ 15-3\lambda \end{pmatrix}$		
	$\overrightarrow{BX} = \begin{pmatrix} 5 \\ -3+\lambda \\ 15-3\lambda \end{pmatrix} - \begin{pmatrix} 11 \\ 9 \\ -7 \end{pmatrix} = \begin{pmatrix} -6 \\ -12+\lambda \\ 22-3\lambda \end{pmatrix}$		
	Method 1		
	$\overrightarrow{BX}\bullet\mathbf{d}_1 = 0 \Rightarrow \begin{pmatrix} -6 \\ -12+\lambda \\ 22-3\lambda \end{pmatrix}\bullet\begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix} = -12+\lambda-66+9\lambda = 0$ leading to $10\lambda - 78 = 0 \Rightarrow \lambda = \dfrac{39}{5}$	(Allow a sign slip in copying $\mathbf{d}_1$) Applies $\overrightarrow{BX}\bullet\mathbf{d}_1 = 0$ and solves the resulting equation to find a value for λ.	M1
	$\overrightarrow{BX} = \begin{pmatrix} -6 \\ -12+\frac{39}{5} \\ 22-3\left(\frac{39}{5}\right) \end{pmatrix} = \begin{pmatrix} -6 \\ -\frac{21}{5} \\ -\frac{7}{5} \end{pmatrix}$	Substitutes their value of λ into their $\overrightarrow{BX}$. **Note:** This mark is dependent upon the previous M1 mark .	dM1
	$d = BX = \sqrt{(-6)^2 + \left(-\frac{21}{5}\right)^2 + \left(-\frac{7}{5}\right)^2} = 7.456540753...$	awrt 7.46	A1
	Method 2		
	Let $\beta = \left\lvert\overrightarrow{BX}\right\rvert^2 = 36+144-24\lambda+\lambda^2+484-132\lambda+9\lambda^2$ $= 10\lambda^2 - 156\lambda + 664$ So $\dfrac{d\beta}{d\lambda} = 20\lambda - 156 = 0 \Rightarrow \lambda = \dfrac{39}{5}$	Finds $\beta = \left\lvert\overrightarrow{BX}\right\rvert^2$ in terms of λ, finds $\dfrac{d\beta}{d\lambda}$ and sets this result equal to 0 and finds a value for λ.	M1
	$\left\lvert\overrightarrow{BX}\right\rvert^2 = 10\left(\frac{39}{5}\right)^2 - 156\left(\frac{39}{5}\right) + 664 = \frac{278}{5}$	Substitutes their value of λ into their $\left\lvert\overrightarrow{BX}\right\rvert^2$. **Note:** This mark is dependent upon the previous M1 mark .	dM1
	$d = BX = \sqrt{\frac{278}{5}} = 7.456540753...$	awrt 7.46	A1

		Question 4 Notes
4. (a)	**M1**	Finds μ and substitutes their μ into l_2
	A1	Point of intersection of $5\mathbf{i}+\mathbf{j}+3\mathbf{k}$. Allow $\begin{pmatrix}5\\1\\3\end{pmatrix}$ or $(5, 1, 3)$.
	Note	You cannot recover the answer for part (a) in part (c) or part (d).
(b)	**M1**	Equates $\mathbf{j}$ components, substitutes their μ and solves to give $\lambda = ...$
	M1	Equates $\mathbf{k}$ components, substitutes their λ and their μ and solves to give $p = ...$ **or** equates $\mathbf{k}$ components to give their "$p - 3\lambda =$ the $\mathbf{k}$ value of A" found in part (b).
	A1	$p = 15$
(c)	**NOTE**	**Part (c) appears as M1A1A1 on ePEN, but now is marked as M1M1A1.**
	M1	Realisation that the dot product is required between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$.
	Note	Allow one slip in candidates copying down their direction vectors, $\mathbf{d}_1$ and $\mathbf{d}_2$.
	dM1	**dependent on the FIRST method mark being awarded.** An attempt to apply the dot product formula between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$.
	A1	anything that rounds to 31.82. This can also be achieved by $180 - 148.1796... =$ awrt 31.82
	Note	$\theta = 0.5553...^c$ is A0.
	Note	**M1A1 for** $\cos\theta = \left(\dfrac{0-16-60}{\sqrt{(0)^2+(4)^2+(-12)^2}\,.\sqrt{(-3)^2+(-4)^2+(5)^2}}\right) = \dfrac{-76}{\sqrt{160}.\sqrt{50}}$

Alternative Method: Vector Cross Product

Only apply this scheme if it is clear that a candidate is applying a vector cross product method.

$\mathbf{d}_1 \times \mathbf{d}_2 = \begin{pmatrix}0\\1\\-3\end{pmatrix} \times \begin{pmatrix}3\\4\\-5\end{pmatrix} = \left\{\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\0 & 1 & -3\\3 & 4 & -5\end{vmatrix} = 7\mathbf{i} - 9\mathbf{j} - 3\mathbf{k}\right\}$	Realisation that the vector cross product is required between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$.	<u>M1</u>
$\sin\theta = \dfrac{\sqrt{(7)^2+(-9)^2+(3)^2}}{\sqrt{(0)^2+(1)^2+(-3)^2}\,.\sqrt{(3)^2+(4)^2+(-5)^2}}$	An attempt to apply the vector cross product formula	dM1 **(A1 on ePEN)**
$\sin\theta = \dfrac{\sqrt{139}}{\sqrt{10}.\sqrt{50}} \Rightarrow \theta = 31.8203116... = 31.82$ (2 dp)	anything that rounds to 31.82	A1

(d)	**M1**	Full method for finding B **and** for finding the magnitude of $\overrightarrow{AB}$ or the magnitude of $\overrightarrow{BA}$.
	dM1	**dependent on the first method mark being awarded.** Writes down correct trigonometric equation involving the shortest distance, d. Eg: $\dfrac{d}{\text{their } AB} = \sin\theta$ **or** $\dfrac{d}{\text{their } AB} = \cos(90-\theta)$, o.e., where " their AB" is a value. and $\theta =$ "their θ" or stated as θ
	A1	anything that rounds to 7.46

Question Number	Scheme		Marks
5.	**Note: You can mark parts (a) and (b) together.**		
(a)	$x = 4t + 3,\ \ y = 4t + 8 + \dfrac{5}{2t}$		
	$\dfrac{dx}{dt} = 4,\ \ \dfrac{dy}{dt} = 4 - \dfrac{5}{2}t^{-2}$	Both $\dfrac{dx}{dt} = 4$ or $\dfrac{dt}{dx} = \dfrac{1}{4}$ and $\dfrac{dy}{dt} = 4 - \dfrac{5}{2}t^{-2}$	B1
	So, $\dfrac{dy}{dx} = \dfrac{4 - \frac{5}{2}t^{-2}}{4}\ \left\{= 1 - \dfrac{5}{8}t^{-2}\ = 1 - \dfrac{5}{8t^2}\right\}$	Candidate's $\dfrac{dy}{dt}$ divided by a candidate's $\dfrac{dx}{dt}$	M1 **o.e.**
	$\{\text{When } t = 2,\}\ \dfrac{dy}{dx} = \dfrac{27}{32}$	$\dfrac{27}{32}$ or 0.84375 **cao**	A1
			[3]
	<u>Way 2: Cartesian Method</u>		
	$\dfrac{dy}{dx} = 1 - \dfrac{10}{(x-3)^2}$	$\dfrac{dy}{dx} = 1 - \dfrac{10}{(x-3)^2}$, simplified or un-simplifed.	B1
		$\dfrac{dy}{dx} = \pm\lambda \pm \dfrac{\mu}{(x-3)^2}$, $\lambda \neq 0, \mu \neq 0$	M1
	$\{\text{When } t = 2,\ x = 11\}\ \dfrac{dy}{dx} = \dfrac{27}{32}$	$\dfrac{27}{32}$ or 0.84375 **cao**	A1
			[3]
	<u>Way 3: Cartesian Method</u>		
	$\dfrac{dy}{dx} = \dfrac{(2x+2)(x-3) - (x^2+2x-5)}{(x-3)^2}$	Correct expression for $\dfrac{dy}{dx}$, simplified or un-simplified.	B1
	$\left\{= \dfrac{x^2 - 6x - 1}{(x-3)^2}\right\}$	$\dfrac{dy}{dx} = \dfrac{f'(x)(x-3) - 1f(x)}{(x-3)^2}$, where $f(x) =$ their "$x^2 + ax + b$", $g(x) = x - 3$	M1
	$\{\text{When } t = 2,\ x = 11\}\ \dfrac{dy}{dx} = \dfrac{27}{32}$	$\dfrac{27}{32}$ or 0.84375 **cao**	A1
			[3]
(b)	$\left\{t = \dfrac{x-3}{4} \Rightarrow\right\}\ y = 4\left(\dfrac{x-3}{4}\right) + 8 + \dfrac{5}{2\left(\dfrac{x-3}{4}\right)}$	Eliminates t to achieve an equation in only x and y	M1
	$y = x - 3 + 8 + \dfrac{10}{x-3}$		
	$y = \dfrac{(x-3)(x-3) + 8(x-3) + 10}{x-3}$ ***or*** $y(x-3) = (x-3)(x-3) + 8(x-3) + 10$ ***or*** $y = \dfrac{(x+5)(x-3) + 10}{x-3}$ ***or*** $y = \dfrac{(x+5)(x-3)}{x-3} + \dfrac{10}{x-3}$	See notes	dM1
	$\Rightarrow y = \dfrac{x^2 + 2x - 5}{x-3}$, $\{a = 2 \text{ and } b = -5\}$	Correct algebra leading to $y = \dfrac{x^2 + 2x - 5}{x-3}$ or $a = 2$ and $b = -5$	A1 **cso**
			[3] **6**

Question Number	Scheme	Marks
5. (b)	<u>**Alternative Method 1 of Equating Coefficients**</u>	
	$y = \dfrac{x^2+ax+b}{x-3} \Rightarrow y(x-3) = x^2+ax+b$	
	$y(x-3) = (4t+3)^2 + 2(4t+3) - 5 = 16t^2+32t+10$	
	$x^2+ax+b = (4t+3)^2 + a(4t+3) + b$	
	$(4t+3)^2 + a(4t+3) + b = 16t^2+32t+10$ — Correct method of obtaining an equation in only t, a and b	M1
	t: $24+4a=32 \Rightarrow a=2$ constant: $9+3a+b=10 \Rightarrow b=-5$ — Equates their coefficients in t and finds both $a = \ldots$ and $b = \ldots$	dM1
	$a=2$ and $b=-5$	A1
		[3]
5. (b)	<u>**Alternative Method 2 of Equating Coefficients**</u>	
	$\left\{t = \dfrac{x-3}{4} \Rightarrow\right\}\ y = 4\left(\dfrac{x-3}{4}\right) + 8 + \dfrac{5}{2\left(\dfrac{x-3}{4}\right)}$ — Eliminates t to achieve an equation in only x and y	M1
	$y = x-3+8+\dfrac{10}{x-3} \Rightarrow y = x+5+\dfrac{10}{(x-3)}$	
	$\underline{\underline{y(x-3)}} = (x+5)(x-3)+10 \Rightarrow x^2+ax+b = \underline{\underline{(x+5)(x-3)+10}}$	dM1
	$\Rightarrow y = \dfrac{x^2+2x-5}{x-3}$ **or** equating coefficients to give $a=2$ and $b=-5$ — Correct algebra leading to $y = \dfrac{x^2+2x-5}{x-3}$ or $a=2$ and $b=-5$	A1 cso
		[3]

		Question 5 Notes
5. (a)	**B1**	$\frac{\mathrm{d}x}{\mathrm{d}t} = 4$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = 4 - \frac{5}{2}t^{-2}$ or $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{8t^2 - 5}{2t^2}$ or $\frac{\mathrm{d}y}{\mathrm{d}t} = 4 - 5(2t)^{-2}(2)$, etc.
	Note	$\frac{\mathrm{d}y}{\mathrm{d}t}$ can be simplified or un-simplified.
	Note	**You can imply the B1 mark by later working.**
	M1	Candidate's $\frac{\mathrm{d}y}{\mathrm{d}t}$ divided by a candidate's $\frac{\mathrm{d}x}{\mathrm{d}t}$ **or** $\frac{\mathrm{d}y}{\mathrm{d}t}$ multiplied by a candidate's $\frac{\mathrm{d}t}{\mathrm{d}x}$
	Note	M1 can be also be obtained by substituting $t = 2$ into both their $\frac{\mathrm{d}y}{\mathrm{d}t}$ and their $\frac{\mathrm{d}x}{\mathrm{d}t}$ and then dividing their values the correct way round.
	A1	$\frac{27}{32}$ or 0.84375 **cao**
(b)	**M1**	Eliminates t to achieve an equation in only x and y.
	dM1	**dependent on the first method mark being awarded.** **Either: (ignoring sign slips or constant slips, noting that k can be 1)** • Combining all three parts of their $\underline{x-3} + \overset{=}{8} + \underline{\underline{\left(\frac{10}{x-3}\right)}}$ to form a single fraction with a common denominator of $\pm k(x-3)$. Accept three separate fractions with the same denominator. • Combining both parts of their $\underline{x+5} + \underline{\underline{\left(\frac{10}{x-3}\right)}}$, (where $\underline{x+5}$ is their $4\left(\frac{x-3}{4}\right) + 8$), to form a single fraction with a common denominator of $\pm k(x-3)$. Accept two separate fractions with the same denominator. • Multiplies both sides of their $y = \underline{x-3} + \overset{=}{8} + \underline{\underline{\left(\frac{10}{x-3}\right)}}$ or their $y = \underline{x+5} + \underline{\underline{\left(\frac{10}{x-3}\right)}}$ by $\pm k(x-3)$. Note that all terms in their equation must be multiplied by $\pm k(x-3)$.
	Note	Condone "invisible" brackets for dM1.
	A1	Correct algebra with no incorrect working leading to $y = \frac{x^2 + 2x - 5}{x - 3}$ or $a = 2$ and $b = -5$
	Note	**Some examples for the award of dM1 in (b):** **dM0** for $y = x - 3 + 8 + \frac{10}{x-3} \rightarrow y = \frac{(x-3)(x-3) + 8 + 10}{x-3}$. Should be $\ldots + 8(x-3) + \ldots$ **dM0** for $y = x - 3 + \frac{10}{x-3} \rightarrow y = \frac{(x-3)(x-3) + 10}{x-3}$. The "8" part has been omitted. **dM0** for $y = x + 5 + \frac{10}{x-3} \rightarrow y = \frac{x(x-3) + 5 + 10}{x-3}$. Should be $\ldots + 5(x-3) + \ldots$ **dM0** for $y = x + 5 + \frac{10}{x-3} \rightarrow y(x-3) = x(x-3) + 5(x-3) + 10(x-3)$. Should be just 10.
	Note	$y = x + 5 + \frac{10}{x-3} \rightarrow y = \frac{x^2 + 2x - 5}{x-3}$ with no intermediate working is dM1A1.

Question Number	Scheme	Marks
6. (a)	$A = \int_0^3 \sqrt{(3-x)(x+1)}\ \mathrm{d}x\ ,\ \ x = 1 + 2\sin\theta$	
	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cos\theta$ — $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cos\theta$ or $2\cos\theta$ used correctly in their working. Can be implied.	B1
	$\left\{\int \sqrt{(3-x)(x+1)}\ \mathrm{d}x \text{ or } \int \sqrt{(3+2x-x^2)}\ \mathrm{d}x\right\}$	
	$= \int \sqrt{\big(3-(1+2\sin\theta)\big)\big((1+2\sin\theta)+1\big)}\ 2\cos\theta\ \{\mathrm{d}\theta\}$ — Substitutes for both x and $\mathrm{d}x$, where $\mathrm{d}x \neq \lambda\mathrm{d}\theta$. Ignore $\mathrm{d}\theta$	M1
	$= \int \sqrt{(2-2\sin\theta)(2+2\sin\theta)}\ 2\cos\theta\ \{\mathrm{d}\theta\}$ $= \int \sqrt{(4-4\sin^2\theta)}\ 2\cos\theta\ \{\mathrm{d}\theta\}$	
	$= \int \sqrt{(4-4(1-\cos^2\theta)}\ 2\cos\theta\ \{\mathrm{d}\theta\}$ or $\int \sqrt{4\cos^2\theta}\ 2\cos\theta\ \{\mathrm{d}\theta\}$ — Applies $\cos^2\theta = 1-\sin^2\theta$ **see notes**	M1
	$= 4\int \cos^2\theta\,\mathrm{d}\theta,\ \{k=4\}$ — $4\int \cos^2\theta\mathrm{d}\theta$ or $\int 4\cos^2\theta\mathrm{d}\theta$ **Note:** $\mathrm{d}\theta$ is required here.	A1
	$0 = 1+2\sin\theta$ or $-1 = 2\sin\theta$ or $\sin\theta = -\frac{1}{2} \Rightarrow \underline{\theta = -\frac{\pi}{6}}$ **and** $3 = 1+2\sin\theta$ or $2 = 2\sin\theta$ or $\sin\theta = 1 \Rightarrow \underline{\theta = \frac{\pi}{2}}$ — See notes	B1
		[5]
(b)	$\left\{k\int \cos^2\theta\ \{\mathrm{d}\theta\}\right\} = \{k\}\int \left(\frac{1+\cos 2\theta}{2}\right)\{\mathrm{d}\theta\}$ — Applies $\cos 2\theta = 2\cos^2\theta - 1$ to their integral	M1
	$= \{k\}\left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right)$ — Integrates to give $\pm\alpha\theta \pm \beta\sin 2\theta,\ \alpha \neq 0,\ \beta \neq 0$ or $k(\pm\alpha\theta \pm \beta\sin 2\theta)$	M1 **(A1 on ePEN)**
	$\left\{\text{So } 4\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta\mathrm{d}\theta = \left[2\theta + \sin 2\theta\right]_{-\frac{\pi}{6}}^{\frac{\pi}{2}}\right\}$ $= \left(2\left(\frac{\pi}{2}\right) + \sin\left(\frac{2\pi}{2}\right)\right) - \left(2\left(-\frac{\pi}{6}\right) + \sin\left(-\frac{2\pi}{6}\right)\right)$	
	$\left\{= (\pi) - \left(-\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)\right\} = \frac{4\pi}{3} + \frac{\sqrt{3}}{2}$ — $\frac{4\pi}{3} + \frac{\sqrt{3}}{2}$ **or** $\frac{1}{6}\left(8\pi + 3\sqrt{3}\right)$	A1 **cao cso**
		[3] **8**

		Question 6 Notes
6. (a)	**B1**	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cos\theta$. Also allow $\mathrm{d}x = 2\cos\theta\,\mathrm{d}\theta$. This mark can be implied by later working.
	Note	You can give B1 for $2\cos\theta$ used correctly in their working.
	M1	Substitutes $x = 1 + 2\sin\theta$ and their $\mathrm{d}x$ $\left(\text{from their rearranged}\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)$ into $\sqrt{(3-x)(x+1)}\,\mathrm{d}x$.
	Note	Condone bracketing errors here.
	Note	$\mathrm{d}x \neq \lambda\mathrm{d}\theta$. For example $\mathrm{d}x \neq \mathrm{d}\theta$.
	Note	Condone substituting $\mathrm{d}x = \cos\theta$ for the 1st M1 after a correct $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cos\theta$ or $\mathrm{d}x = 2\cos\theta\mathrm{d}\theta$
	M1	Applies either • $1 - \sin^2\theta = \cos^2\theta$ • $\lambda - \lambda\sin^2\theta$ or $\lambda(1-\sin^2\theta) = \lambda\cos^2\theta$ • $4 - 4\sin^2\theta = 4 + 2\cos 2\theta - 2 = 2 + 2\cos 2\theta = 4\cos^2\theta$ to their expression where λ is a numerical value.
	A1	Correctly proves that $\int\sqrt{(3-x)(x+1)}\,\mathrm{d}x$ is equal to $4\int\cos^2\theta\mathrm{d}\theta$ or $\int 4\cos^2\theta\mathrm{d}\theta$
	Note	All three previous marks must have been awarded before A1 can be awarded.
	Note	Their final answer must include $\mathrm{d}\theta$.
	Note	You can ignore limits for the final A1 mark.
	B1	Evidence of a correct equation in $\sin\theta$ or $\sin^{-1}\theta$ for both *x*-values leading to both θ values. Eg: • $0 = 1 + 2\sin\theta$ or $-1 = 2\sin\theta$ or $\sin\theta = -\frac{1}{2}$ which then leads to $\theta = -\frac{\pi}{6}$, **and** • $3 = 1 + 2\sin\theta$ or $2 = 2\sin\theta$ or $\sin\theta = 1$ which then leads to $\theta = \frac{\pi}{2}$
	Note	Allow B1 for $x = 1 + 2\sin\left(-\frac{\pi}{6}\right) = 0$ **and** $x = 1 + 2\sin\left(\frac{\pi}{2}\right) = 3$
	Note	Allow B1 for $\sin\theta = \left(\frac{x-1}{2}\right)$ or $\theta = \sin^{-1}\left(\frac{x-1}{2}\right)$ followed by $x = 0,\ \theta = -\frac{\pi}{6};\ x = 3,\ \theta = \frac{\pi}{2}$
(b)	**NOTE**	**Part (b) appears as M1A1A1 on ePEN, but is now marked as M1M1A1.**
	M1	Writes down a correct equation involving $\cos 2\theta$ and $\cos^2\theta$ **Eg:** $\cos 2\theta = 2\cos^2\theta - 1$ or $\cos^2\theta = \frac{1+\cos 2\theta}{2}$ or $\lambda\cos^2\theta = \lambda\left(\frac{1+\cos 2\theta}{2}\right)$ and ***applies*** it to their integral. **Note:** Allow M1 for a correctly stated formula (via an incorrect rearrangement) being applied to their integral.
	M1	Integrates to give an expression of the form $\pm\alpha\theta \pm \beta\sin 2\theta$ or $k(\pm\alpha\theta \pm \beta\sin 2\theta)$, $\alpha \neq 0, \beta \neq 0$ (can be simplified or un-simplified).
	A1	A ***correct solution in part (b)*** leading to a "two term" exact answer. Eg: $\frac{4\pi}{3} + \frac{\sqrt{3}}{2}$ **or** $\frac{8\pi}{6} + \frac{\sqrt{3}}{2}$ **or** $\frac{1}{6}\left(8\pi + 3\sqrt{3}\right)$
	Note	5.054815… from no working is M0M0A0.
	Note	Candidates can work in terms of *k* (note that *k* is not given in (a)) for the M1M1 marks in part (b).
	Note	If they incorrectly obtain $4\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}}\cos^2\theta\mathrm{d}\theta$ in part (a) (or guess $k = 4$) then the final A1 is available for a correct solution in part (b) only.

Question Number	Scheme		Marks
7. (a)	$\dfrac{2}{P(P-2)} = \dfrac{A}{P} + \dfrac{B}{(P-2)}$		
	$2 \equiv A(P-2) + BP$	Can be implied.	M1
	$A = -1,\ B = 1$	Either one.	A1
	giving $\dfrac{1}{(P-2)} - \dfrac{1}{P}$	See notes. **cao, aef**	A1
			[3]
(b)	$\dfrac{\mathrm{d}P}{\mathrm{d}t} = \dfrac{1}{2}P(P-2)\cos 2t$		
	$\displaystyle\int \frac{2}{P(P-2)}\,\mathrm{d}P = \int \cos 2t\ \mathrm{d}t$	can be implied by later working	B1 oe
	$\ln(P-2) - \ln P = \dfrac{1}{2}\sin 2t\ \ (+c)$	$\pm\lambda\ln(P-2) \pm \mu\ln P$, $\lambda \neq 0,\ \mu \neq 0$	M1
		$\ln(P-2) - \ln P = \dfrac{1}{2}\sin 2t$	A1
	$\{t = 0,\ P = 3 \Rightarrow\}\quad \ln 1 - \ln 3 = 0 + c \quad \{\Rightarrow\ c = -\ln 3 \text{ or } \ln(\tfrac{1}{3})\}$	See notes	M1
	$\ln(P-2) - \ln P = \dfrac{1}{2}\sin 2t - \ln 3$ $\ln\left(\dfrac{3(P-2)}{P}\right) = \dfrac{1}{2}\sin 2t$		
	$\dfrac{3(P-2)}{P} = \mathrm{e}^{\frac{1}{2}\sin 2t}$	Starting from an equation of the form $\pm\lambda\ln(P-\beta) \pm \mu\ln P = \pm K\sin\delta t + c$, $\lambda,\mu,\beta,K,\delta \neq 0$, applies a fully correct method to eliminate their logarithms. **Must have a constant of integration that need not be evaluated (see note)**	M1
	$3(P-2) = P\mathrm{e}^{\frac{1}{2}\sin 2t} \Rightarrow 3P - 6 = P\mathrm{e}^{\frac{1}{2}\sin 2t}$ gives $3P - P\mathrm{e}^{\frac{1}{2}\sin 2t} = 6 \Rightarrow P(3 - \mathrm{e}^{\frac{1}{2}\sin 2t}) = 6$	A complete method of rearranging to make P the subject. **Must have a constant of integration that need not be evaluated (see note)**	dM1
	$P = \dfrac{6}{(3 - \mathrm{e}^{\frac{1}{2}\sin 2t})}$ *	Correct proof.	A1 * cso
			[7]
(c)	$\{\text{population} = 4000 \Rightarrow\}\ P = 4$	States $P = 4$ or applies $P = 4$	M1
	$\dfrac{1}{2}\sin 2t = \ln\left(\dfrac{3(4-2)}{4}\right)\ \left\{= \ln\left(\dfrac{3}{2}\right)\right\}$	Obtains $\pm\lambda\sin 2t = \ln k$ or $\pm\lambda\sin t = \ln k$, $\lambda \neq 0,\ k > 0$ where λ and k are numerical values and λ can be 1	M1
	$t = 0.4728700467...$	anything that rounds to 0.473 Do not apply isw here	A1
			[3] **13**

Question Number	Scheme		Marks
	<u>Method 2 for Q7(b)</u>		
7. (b)	$\ln(P-2) - \ln P = \frac{1}{2}\sin 2t \;\; (+c)$	As before for…	**B1M1A1**
	$\ln\left(\frac{(P-2)}{P}\right) = \frac{1}{2}\sin 2t + c$		
	$\frac{(P-2)}{P} = \mathrm{e}^{\frac{1}{2}\sin 2t + c}$ or $\frac{(P-2)}{P} = A\mathrm{e}^{\frac{1}{2}\sin 2t}$	Starting from an equation of the form $\pm\lambda\ln(P-\beta) \pm \mu\ln P = \pm K\sin\delta t + c$, $\lambda, \mu, \beta, K, \delta \neq 0$, applies a fully correct method to eliminate their logarithms. **Must have a constant of integration that need not be evaluated (see note)**	**3rd M1**
	$(P-2) = AP\mathrm{e}^{\frac{1}{2}\sin 2t} \Rightarrow P - AP\mathrm{e}^{\frac{1}{2}\sin 2t} = 2$ $\Rightarrow P(1 - A\mathrm{e}^{\frac{1}{2}\sin 2t}) = 2 \Rightarrow P = \frac{2}{(1 - A\mathrm{e}^{\frac{1}{2}\sin 2t})}$	A complete method of rearranging to make P the subject. Condone sign slips or constant errors. **Must have a constant of integration that need not be evaluated (see note)**	**4th dM1**
	$\{t = 0, P = 3 \Rightarrow\} \quad 3 = \frac{2}{(1 - A\mathrm{e}^{\frac{1}{2}\sin 2(0)})}$	See notes **(Allocate this mark as the 2nd M1 mark on ePEN).**	**2nd M1**
	$\left\{\Rightarrow \;\; 3 = \frac{2}{(1-A)} \Rightarrow A = \frac{1}{3}\right\}$		
	$\Rightarrow P = \frac{2}{\left(1 - \frac{1}{3}\mathrm{e}^{\frac{1}{2}\sin 2t}\right)} \Rightarrow P = \frac{6}{(3 - \mathrm{e}^{\frac{1}{2}\sin 2t})}$ *	Correct proof.	**A1 * cso**

Question 7 Notes

7. (a)	**M1**	Forming a correct identity. For example, $2 \equiv A(P-2) + BP$ from $\frac{2}{P(P-2)} = \frac{A}{P} + \frac{B}{(P-2)}$
	Note	A and B are not referred to in question.
	A1	Either one of $A = -1$ or $B = 1$.
	A1	$\frac{1}{(P-2)} - \frac{1}{P}$ or any equivalent form. This answer ***cannot*** be recovered from part (b).
	Note	M1A1A1 can also be given for a candidate who finds both $A = -1$ and $B = 1$ and $\frac{A}{P} + \frac{B}{(P-2)}$ is seen in their working.
	Note	Candidates can use 'cover-up' rule to write down $\frac{1}{(P-2)} - \frac{1}{P}$, so as to gain all three marks.
	Note	Equating coefficients from $2 \equiv A(P-2) + BP$ gives $A + B = 2, -2A = 2 \Rightarrow A = -1, \; B = 1$

7. (b)	**B1**	Separates variables as shown on the Mark Scheme. $\mathrm{d}P$ and $\mathrm{d}t$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.
	Note	**Eg:** $\int \frac{2}{P^2 - 2P}\,\mathrm{d}P = \int \cos 2t\,\mathrm{d}t$ or $\int \frac{1}{P(P-2)}\,\mathrm{d}P = \frac{1}{2}\int \cos 2t\,\mathrm{d}t$ **o.e.** are also fine for B1.
	1st M1	$\pm\lambda\ln(P-2) \pm \mu\ln P,\ \lambda \neq 0,\ \mu \neq 0$. Also allow $\pm\lambda\ln(M(P-2)) \pm \mu\ln NP;\ M, N$ can be 1.
	Note	Condone $2\ln(P-2) + 2\ln P$ or $2\ln(P(P-2))$ or $2\ln(P^2 - 2P)$ or $\ln(P^2 - 2P)$
	1st A1	Correct result of $\ln(P-2) - \ln P = \frac{1}{2}\sin 2t$ or $2\ln(P-2) - 2\ln P = \sin 2t$ o.e. with or without $+c$
	2nd M1	Some evidence of using both $t = 0$ and $P = 3$ in an integrated equation containing a constant of integration. Eg: c or A, etc.
	3rd M1	Starting from an equation of the form $\pm\lambda\ln(P-\beta) \pm \mu\ln P = \pm K\sin\delta t + c,\ \lambda,\mu,\beta,K,\delta \neq 0$, applies a fully correct method to eliminate their logarithms.
	4th M1	**dependent on the third method mark being awarded.** A complete method of rearranging to make *P* the subject. Condone sign slips or constant errors.
	Note	For the 3rd M1 and 4th M1 marks, a candidate needs to have included a constant of integration, in their working. eg. $c, A, \ln A$ or an evaluated constant of integration.
	2nd A1	Correct proof of $P = \dfrac{6}{(3 - \mathrm{e}^{\frac{1}{2}\sin 2t})}$. **Note:** This answer is given in the question.
	Note	$\ln\left(\frac{(P-2)}{P}\right) = \frac{1}{2}\sin 2t + c$ followed by $\frac{(P-2)}{P} = \mathrm{e}^{\frac{1}{2}\sin 2t} + \mathrm{e}^c$ is 3rd M0, 4th M0, 2nd A0.
	Note	$\ln\left(\frac{(P-2)}{P}\right) = \frac{1}{2}\sin 2t + c \to \frac{(P-2)}{P} = \mathrm{e}^{\frac{1}{2}\sin 2t + c} \to \frac{(P-2)}{P} = \mathrm{e}^{\frac{1}{2}\sin 2t} + \mathrm{e}^c$ is final M1M0A0
	<u>4th M1 for making *P* the subject</u> **Note there are three type of manipulations here which are considered acceptable for making *P* the subject.** **(1)** M1 for $\frac{3(P-2)}{P} = \mathrm{e}^{\frac{1}{2}\sin 2t} \Rightarrow 3(P-2) = P\mathrm{e}^{\frac{1}{2}\sin 2t} \Rightarrow 3P - 6 = P\mathrm{e}^{\frac{1}{2}\sin 2t} \Rightarrow P(3 - \mathrm{e}^{\frac{1}{2}\sin 2t}) = 6$ $\Rightarrow P = \frac{6}{(3 - \mathrm{e}^{\frac{1}{2}\sin 2t})}$ **(2)** M1 for $\frac{3(P-2)}{P} = \mathrm{e}^{\frac{1}{2}\sin 2t} \Rightarrow 3 - \frac{6}{P} = \mathrm{e}^{\frac{1}{2}\sin 2t} \Rightarrow 3 - \mathrm{e}^{\frac{1}{2}\sin 2t} = \frac{6}{P} \Rightarrow\Rightarrow P = \frac{6}{(3 - \mathrm{e}^{\frac{1}{2}\sin 2t})}$ **(3)** M1 for $\left\{\ln(P-2) + \ln P = \frac{1}{2}\sin 2t + \ln 3 \Rightarrow\right\} P(P-2) = 3\mathrm{e}^{\frac{1}{2}\sin 2t} \Rightarrow P^2 - 2P = 3\mathrm{e}^{\frac{1}{2}\sin 2t}$ $\Rightarrow (P-1)^2 - 1 = 3\mathrm{e}^{\frac{1}{2}\sin 2t}$ leading to $P = ..$	
(c)	**M1**	States $P = 4$ or applies $P = 4$
	M1	Obtains $\pm\lambda\sin 2t = \ln k$ or $\pm\lambda\sin t = \ln k$, where λ and k are numerical values and λ can be 1
	A1	anything that rounds to 0.473. ***(Do not apply isw here)***
	Note	***Do not apply ignore subsequent working for A1.*** (Eg: 0.473 followed by 473 years is A0.)
	Note	<u>Use of $P = 4000$</u>: Without the mention of $P = 4$, $\frac{1}{2}\sin 2t = \ln 2.9985$ or $\sin 2t = 2\ln 2.9985$ or $\sin 2t = 2.1912...$ will usually imply M0M1A0
	Note	<u>Use of Degrees:</u> $t =$ awrt 27.1 will usually imply M1M1A0

Question Number	Scheme		Marks
8. (a)	$\left\{y = 3^x \Rightarrow\right\}\ \dfrac{dy}{dx} = 3^x \ln 3$	$\dfrac{dy}{dx} = 3^x \ln 3$ or $\ln 3\left(e^{x\ln 3}\right)$ or $y\ln 3$	B1
	Either T: $y - 9 = 3^2 \ln 3(x-2)$ **or T:** $y = (3^2\ln 3)x + 9 - 18\ln 3$, where $9 = (3^2\ln 3)(2) + c$	See notes	M1
	$\{$Cuts x-axis $\Rightarrow y = 0 \Rightarrow\}$		
	$-9 = 9\ln 3(x-2)$ or $0 = (3^2\ln 3)x + 9 - 18\ln 3$,	Sets $y = 0$ in their tangent equation and progresses to $x = ...$	M1
	So, $x = 2 - \dfrac{1}{\ln 3}$	$2 - \dfrac{1}{\ln 3}$ or $\dfrac{2\ln 3 - 1}{\ln 3}$ o.e.	A1 **cso**
			[4]
(b)	$V = \pi\displaystyle\int \left(3^x\right)^2 \{dx\}$ or $\pi\displaystyle\int 3^{2x}\{dx\}$ or $\pi\displaystyle\int 9^x\{dx\}$	$V = \pi\displaystyle\int \left(3^x\right)^2$ with or without dx, which can be implied	B1 o.e.
	$= \{\pi\}\left(\dfrac{3^{2x}}{2\ln 3}\right)$ or $= \{\pi\}\left(\dfrac{9^x}{\ln 9}\right)$	Eg: either $3^{2x} \to \dfrac{3^{2x}}{\pm\alpha(\ln 3)}$ or $\pm\alpha(\ln 3)3^{2x}$ or $9^x \to \dfrac{9^x}{\pm\alpha(\ln 9)}$ or $\pm\alpha(\ln 9)9^x$, $\underline{\underline{\alpha \in}}$	M1
		$3^{2x} \to \dfrac{3^{2x}}{2\ln 3}$ or $9^x \to \dfrac{9^x}{\ln 9}$ or $e^{2x\ln 3} \to \dfrac{1}{2\ln 3}\left(e^{2x\ln 3}\right)$	A1 o.e.
	$\left\{V = \pi\displaystyle\int_0^2 3^{2x}\,dx = \{\pi\}\left[\dfrac{3^{2x}}{2\ln 3}\right]_0^2\right\} = \{\pi\}\left(\dfrac{3^4}{2\ln 3} - \dfrac{1}{2\ln 3}\right)\ \left\{= \dfrac{40\pi}{\ln 3}\right\}$	**Dependent on the previous method mark.** Substitutes $x = 2$ and $x = 0$ and subtracts the correct way round.	dM1
	$V_{cone} = \dfrac{1}{3}\pi(9)^2\left(\dfrac{1}{\ln 3}\right)\ \left\{= \dfrac{27\pi}{\ln 3}\right\}$	$V_{cone} = \dfrac{1}{3}\pi(9)^2(2 - \text{their } (a))$. See notes.	B1ft
	$\left\{\text{Vol}(S) = \dfrac{40\pi}{\ln 3} - \dfrac{27\pi}{\ln 3}\right\} = \underline{\dfrac{13\pi}{\ln 3}}$	$\dfrac{13\pi}{\ln 3}$ or $\dfrac{26\pi}{\ln 9}$ or $\dfrac{26\pi}{2\ln 3}$ etc., isw	A1 o.e.
		$\{\text{Eg: } p = 13\pi,\ q = \ln 3\}$	**[6]** **10**
(b)	**<u>Alternative Method 1: Use of a substitution</u>**		
	$V = \pi\displaystyle\int \left(3^x\right)^2\{dx\}$		B1 o.e.
	$\left\{u = 3^x \Rightarrow \dfrac{du}{dx} = 3^x\ln 3 = u\ln 3\right\}$ $V = \{\pi\}\displaystyle\int \dfrac{u^2}{u\ln 3}\{du\} = \{\pi\}\displaystyle\int \dfrac{u}{\ln 3}\{du\}$		
	$= \{\pi\}\left(\dfrac{u^2}{2\ln 3}\right)$	$\left(3^x\right)^2 \to \dfrac{u^2}{\pm\alpha(\ln 3)}$ or $\pm\alpha(\ln 3)u^2$, where $u = 3^x$	M1
		$\left(3^x\right)^2 \to \dfrac{u^2}{2(\ln 3)}$, where $u = 3^x$	A1
	$\left\{V = \pi\displaystyle\int_0^2 \left(3^x\right)^2 dx = \{\pi\}\left[\dfrac{u^2}{2\ln 3}\right]_1^9\right\} = \{\pi\}\left(\dfrac{9^2}{2\ln 3} - \dfrac{1}{2\ln 3}\right)\ \left\{= \dfrac{40\pi}{\ln 3}\right\}$	Substitutes limits of 9 and 1 in u (or 2 and 0 in x) and subtracts the correct way round.	dM1
	then apply the main scheme.		

	Question 8 Notes	
8. (a)	**B1**	$\frac{dy}{dx} = 3^x \ln 3$ or $\ln 3\left(e^{x\ln 3}\right)$ or $y\ln 3$. Can be implied by later working.
	M1	Substitutes either $x = 2$ **or** $y = 9$ into their $\frac{dy}{dx}$ which is a function of *x* or *y* to find m_T **and** • **either** applies $y - 9 = (\text{their } m_T)(x - 2)$, where m_T is a numerical value. • **or** applies $y = (\text{their } m_T)x + \text{their } c$, where m_T is a numerical value and c is found by solving $9 = (\text{their } m_T)(2) + c$
	Note	The first M1 mark can be implied from later working.
	M1	Sets $y = 0$ in their ***tangent*** equation, where m_T is a numerical value, (seen or implied) and progresses to $x = ...$
	A1	An exact value of $2 - \frac{1}{\ln 3}$ or $\frac{2\ln 3 - 1}{\ln 3}$ or $\frac{\ln 9 - 1}{\ln 3}$ by a correct solution only.
	Note	Allow A1 for $2 - \frac{\lambda}{\lambda \ln 3}$ or $\frac{\lambda(2\ln 3 - 1)}{\lambda \ln 3}$ or $\frac{\lambda(\ln 9 - 1)}{\lambda \ln 3}$ or $2 - \frac{\lambda}{\lambda \ln 3}$, where λ is an integer, and ignore subsequent working.
	Note	Using a changed gradient (i.e. applying $\frac{-1}{\text{their } \frac{dy}{dx}}$ or $\frac{1}{\text{their } \frac{dy}{dx}}$) is M0 M0 in part (a).
	Note	Candidates who invent a value for m_T (which bears no resemblance to their gradient function) **cannot** gain the 1st M1 and 2nd M1 mark in part (a).
	Note	A decimal answer of 1.089760773… (without a correct **exact** answer) is A0.
8. (b)	**B1**	A correct expression for the volume with or without dx
	Note	**Eg:** Allow B1 for $\pi\int \left(3^x\right)^2 \{dx\}$ or $\pi\int 3^{2x}\{dx\}$ or $\pi\int 9^x\{dx\}$ or $\pi\int \left(e^{x\ln 3}\right)^2\{dx\}$ or $\pi\int \left(e^{2x\ln 3}\right)\{dx\}$ or $\pi\int e^{x\ln 9}\{dx\}$ with or without dx
	M1	Either $3^{2x} \to \frac{3^{2x}}{\pm\alpha(\ln 3)}$ or $\pm\alpha(\ln 3)3^{2x}$ or $9^x \to \frac{9^x}{\pm\alpha(\ln 9)}$ or $\pm\alpha(\ln 9)9^x$ $e^{2x\ln 3} \to \frac{e^{2x\ln 3}}{\pm\alpha(\ln 3)}$ or $\pm\alpha(\ln 3)e^{2x\ln 3}$ or $e^{x\ln 9} \to \frac{e^{x\ln 9}}{\pm\alpha(\ln 9)}$ or $\pm\alpha(\ln 9)e^{x\ln 9}$, etc where $\alpha \in \frown$
	Note	$3^{2x} \to \frac{3^{2x+1}}{\pm\alpha(\ln 3)}$ or $9^x \to \frac{9^{x+1}}{\pm\alpha(\ln 3)}$ are allowed for M1
	Note	$3^{2x} \to \frac{3^{2x+1}}{2x+1}$ or $9^x \to \frac{9^{x+1}}{x+1}$ are both M0
	Note	M1 can be given for $9^{2x} \to \frac{9^{2x}}{\pm\alpha(\ln 9)}$ or $\pm\alpha(\ln 9)9^{2x}$
	A1	Correct integration of 3^{2x}. Eg: $3^{2x} \to \frac{3^{2x}}{2\ln 3}$ or $\frac{3^{2x}}{\ln 9}$ or $9^x \to \frac{9^x}{\ln 9}$ or $e^{2x\ln 3} \to \frac{1}{2\ln 3}\left(e^{2x\ln 3}\right)$
	dM1	**dependent on the previous method mark being awarded.** Attempts to apply $x = 2$ and $x = 0$ to integrated expression and subtracts the correct way round.
	Note	Evidence of a proper consideration of the limit of 0 is needed for M1. So subtracting 0 is M0.

	dM1	**dependent on the previous method mark being awarded.** Attempts to apply $x = 2$ and $x = 0$ to integrated expression and subtracts the correct way round.
	Note	Evidence of a proper consideration of the limit of 0 is needed for M1. So subtracting 0 is M0.
	B1ft	$V_{\text{cone}} = \frac{1}{3}\pi(9)^2\left(2 - \text{their answer to part } (a)\right)$. Sight of $\frac{27\pi}{\ln 3}$ implies the B1 mark.
	Note	Alternatively they can apply the volume formula to the line segment. They need to achieve the result highlighted by ******** on either page 29 or page 30 in order to obtain the B1ft mark.
	A1	$\frac{13\pi}{\ln 3}$ or $\frac{26\pi}{\ln 9}$ or $\frac{26\pi}{2\ln 3}$, etc. , where their answer is in the form $\frac{p}{q}$
	Note	The π in the volume formula is only needed for the 1st B1 mark and the final A1 mark.
	Note	A decimal answer of 37.17481128… (without a correct **exact** answer) is A0.
	Note	A candidate who applies $\int 3^x \mathrm{d}x$ will either get B0 M0 A0 M0 **B0** A0 **or** B0 M0 A0 M0 **B1** A0
	Note	$\pi\int 3^{x^2}\,\mathrm{d}x$ unless recovered is B0.
	Note	<u>**Be careful! A correct answer may follow from incorrect working**</u> $V = \pi\int_0^2 3^{x^2}\,\mathrm{d}x - \frac{1}{3}\pi(9)^2\left(\frac{1}{\ln 3}\right) = \pi\left[\frac{3^{x^2}}{2\ln 3}\right]_0^2 - \frac{27\pi}{\ln 3} = \frac{\pi 3^4}{2\ln 3} - \frac{\pi}{2\ln 3} - \frac{27\pi}{\ln 3} = \frac{13\pi}{\ln 3}$ would score B0 M0 A0 dM0 M1 A0.
8. (b)	<u>**2nd B1ft mark for finding the Volume of a Cone**</u>	

$$V_{\text{cone}} = \pi\int_{2-\frac{1}{\ln 3}}^{2} \left(9x\ln 3 - 18\ln 3 + 9\right)^2 \mathrm{d}x$$

$$= \pi\left[\frac{\left(9x\ln 3 - 18\ln 3 + 9\right)^3}{27\ln 3}\right]_{2-\frac{1}{\ln 3}\text{ or their part (a) answer}}^{2} \qquad ****$$

Award B1ft here where their lower limit is $2 - \frac{1}{\ln 3}$ or their part (a) answer.

$$= \pi\left(\left(\frac{\left(18\ln 3 - 18\ln 3 + 9\right)^3}{27\ln 3}\right) - \left(\frac{\left(9\left(2 - \frac{1}{\ln 3}\right)\ln 3 - 18\ln 3 + 9\right)^3}{27\ln 3}\right)\right)$$

$$= \pi\left(\left(\frac{729}{27\ln 3}\right) - \left(\frac{\left(18\ln 3 - 9 - 18\ln 3 + 9\right)^3}{27\ln 3}\right)\right)$$

$$= \frac{27\pi}{\ln 3}$$

8. (b)	**2nd B1ft mark for finding the Volume of a Cone** **Alternative method 2:** $V_{\text{cone}} = \pi \int_{2-\frac{1}{\ln 3}}^{2} (9x\ln 3 - 18\ln 3 + 9)^2 \, \mathrm{d}x$ $= \pi \int_{2-\frac{1}{\ln 3}}^{2} \left(81x^2(\ln 3)^2 - 324x(\ln 3)^2 + 162x\ln 3 - 324\ln 3 + 324(\ln 3)^2 + 81\right) \mathrm{d}x$ $= \pi \left[27x^3(\ln 3)^2 - 162x^2(\ln 3)^2 + 81x^2\ln 3 - 324x\ln 3 + 324x(\ln 3)^2 + 81x\right]_{2-\frac{1}{\ln 3}}^{2}$ Award B1ft here where their lower limit is $2 - \dfrac{1}{\ln 3}$ or their part (a) answer. **** $= \pi\left(\left(216(\ln 3)^2 - 648(\ln 3)^2 + 324\ln 3 - 648\ln 3 + 648(\ln 3)^2 + 162\right) - \left(27\left(2-\frac{1}{\ln 3}\right)^3(\ln 3)^2 - 162\left(2-\frac{1}{\ln 3}\right)^2(\ln 3)^2 + 81\left(2-\frac{1}{\ln 3}\right)^2\ln 3 - 324\left(2-\frac{1}{\ln 3}\right)\ln 3 + 324\left(2-\frac{1}{\ln 3}\right)(\ln 3)^2 + 81\left(2-\frac{1}{\ln 3}\right)\right)\right)$ $= \pi\left(\left(216(\ln 3)^2 - 324\ln 3 + 162\right) - \left(27\left(8 - \frac{12}{\ln 3} + \frac{6}{(\ln 3)^2} - \frac{1}{(\ln 3)^3}\right)(\ln 3)^2 - 162\left(4 - \frac{4}{\ln 3} + \frac{1}{(\ln 3)^2}\right)(\ln 3)^2 + 81\left(4 - \frac{4}{\ln 3} + \frac{1}{(\ln 3)^2}\right)\ln 3 - 324\left(2-\frac{1}{\ln 3}\right)\ln 3 + 324\left(2-\frac{1}{\ln 3}\right)(\ln 3)^2 + 81\left(2-\frac{1}{\ln 3}\right)\right)\right)$ $= \pi\left(\left(216(\ln 3)^2 - 324\ln 3 + 162\right) - \left(216(\ln 3)^2 - 324\ln 3 + 162 - \frac{27}{\ln 3} - 648(\ln 3)^2 + 648\ln 3 - 162 + 324\ln 3 - 324 + \frac{81}{\ln 3} - 648\ln 3 + 324 + 648(\ln 3)^2 - 324\ln 3 + 162 - \frac{81}{\ln 3}\right)\right)$ $= \pi\left(\left(216(\ln 3)^2 - 324\ln 3 + 162\right) - \left(216(\ln 3)^2 - 324\ln 3 + 162 - \frac{27}{\ln 3}\right)\right)$ $= \dfrac{27\pi}{\ln 3}$